SFL Volume 31 (2022-Q3)

[2022-07-01.14:25 Fri> At last, an ugly, messy proof that age slope = minus the velocity

For the figure I'm doing:

$$a/b = v \qquad \sqrt{a^2+b^2} = D/2 \qquad \text{Solve for } b \text{ by eliminating } a$$

$$a = bv \qquad \sqrt{(bv)^2+b^2} = D/2 \qquad b\sqrt{v^2+1} = D/2 \qquad b = \frac{D}{2\sqrt{v^2+1}}$$

$$D = \sqrt{\frac{1+v^2}{1-v^2}} \qquad \text{so} \qquad \boldsymbol{b} = \frac{\sqrt{\frac{1+v^2}{1-v^2}}}{2\sqrt{1+v^2}} = ½\frac{1}{\sqrt{1-v^2}} = \frac{\gamma}{2} \quad = \quad ½\boldsymbol{\gamma}$$

Also:

$$\frac{\gamma}{2} - (\gamma-1) = \frac{\gamma}{2} - \gamma + 1 = 1 - \frac{\gamma}{2}$$

And wow, that is *so* wrong — how did I get the side of the equilateral square triangle at the end of a clock box as $\gamma - 1$, which is patently absurd as gamma gets big? Ah: Back on page [2022-07-01.15:46 Thu] I used the *coincidence* that this is true for the single case of R=2. Since that's a reference figure and it's just a deletion, I'll red-line the problem here and delete the R=2 only equation from the PPT and from the Word inline image of it.

Starting over on that part:

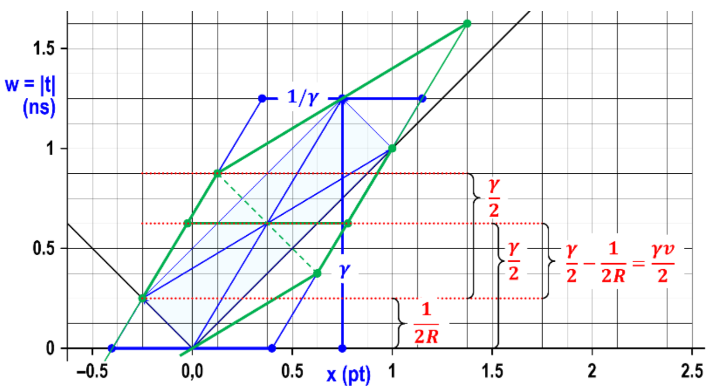
$$\frac{\gamma}{2} - \frac{1}{2R} \quad = \quad \frac{\gamma}{2} - \frac{1}{2\left(\sqrt{\frac{\gamma+\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}}}\right)} \quad = \quad ½\left(\gamma - \sqrt{\frac{\gamma+\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}}}\right)$$

That's messy. Let's try velocity:

$$\frac{\gamma}{2} - \frac{1}{2R} \quad = \quad ½\left(1\Big/\sqrt{\frac{1+v}{1-v}}\right) \quad = \quad ½\left(\frac{1}{\sqrt{1-v^2}} - \frac{\sqrt{1-v}}{\sqrt{1+v}}\right) \quad = \quad ½\left(\frac{1}{\sqrt{1-v}\ \sqrt{1+v}} - \frac{\sqrt{1-v}}{\sqrt{1+v}}\right)$$

$$= \quad ½\left(\frac{1-(1-v)}{\sqrt{1-v}\ \sqrt{1+v}}\right) \quad = \quad ½\frac{v}{\sqrt{1-v^2}} \quad = \quad ½\boldsymbol{\gamma v}$$

The final ratio in the figure is $(½\gamma v)/(½\gamma) = v$. So wow, yes, the time slope is $-v$.



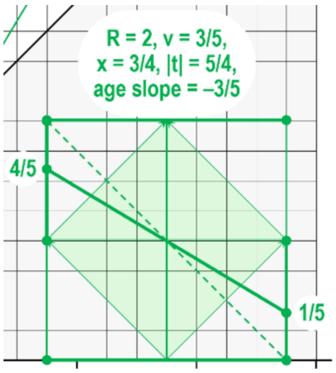
That's nice etc., etc., but it's also about as needlessly messy of a proof of it as I can think of, what with all of the odd roots popping up by projecting the various triangles onto w vertical in the figure. So still, I wonder: What is the likely insanely simple point I keep missing that makes the age slope into minus the velocity?

[2022-07-01.17:03 Fri]

[2022-07-01.17:08 Fri> The more interesting time slope is the Lorentz compressed one

So I guess the age slope need its own letter, though even more blatantly than with all the other constants, it's just a simple transform of unitless velocity v? *Maybe* alpha, $\alpha = -v$?

Is that even worth an entry in the table? No, I'm not doing it! It's the negative velocity!



Also, before I start defining still more dimensionless transforms of the velocity, I note that in defining the slope I've so far skipped quite glibly over the point that this is the *unobservable* time slope that goes on *inside* the moving clock, as with the $v=0.6$ example at left. While the resulting slope of up to -1 — forty-five degrees tilt down — looks pretty impressive in the unitless, 1-ns-equals-1-photon-foot coordinates of my figures, the sad truth is that in a 100 phoot spaceship traveling at 99.99% of c the age slope, which is *always unmeasurable* from within the frame to which it applies, add up to a whopping -100 nanoseconds. Even if the ship *could* figure out a way to know that the clocks at the stern are set 100 ns in the future from clocks in its bow, relative to rest frame, a 100 ns time delta from stern to bow is not exactly going to endanger most forms of timekeeping. It could mess royally with fast networks, but again, that's *if it could be detected*, which it cannot. The rest-frame's already-finished past back at the bow and its yet-to-be future at the stern both are delayed *just enough* in any attempt to compare them to ensure that anyone in the moving frame of the shop cannot tell any causal problem or discrepancy. SR rocks!

Where the time slope *is* observable is in the rest frame, where it is Lorentz contracted and thus can go to infinity:

$$\alpha = v\gamma$$

I'll dive more into this detectable version of alpha later [2022-07-01.17:35 Fri]

[2022-07-02.17:22 Sat> The rest-frame age gradient matches the x distance traveled

So, I was thinking even as I wrote down $\alpha = v\gamma$ that it sounded familiar from my earlier table — didn't I have some entry that was just v times gamma? Well, yes, but it certainly was not what I was expecting intuitively: It's just *distance traveled*. Eh??

In unitless velocities, gamma is just a stand-in for the time until completion of one cycle of the moving clock. Thus is makes perfect sense that multiplying gamma by velocity in this specific context of comparing two clock cycles gives the distance traveled by the moving clock. I'm just not used to thinking of gamma in that fashion?

But still… just how weird and counterintuitive *is* that? As with any Lorentz-multiplied quantity, the moving-frame and rest-frame age gradients (time slopes? which is it, Terry?) are quite close to each other for human-range velocities that are a tiny fraction of light speed. But for $v=0.6$ it's 0.75, and for $v=0.999$ it's 22.3. [2022-07-02.17:39 Sat]